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Superprism effect in bidimensional rectangular photonic crystals

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In this letter, we show that photonic crystals with geometries of lower symmetry, such as the rectangular geometry, are uniquely suited for applications involving the superprism effect. The extra degree of freedom provided by the anisotropy of the unit cell allows more freedom in searching for suitable iso-frequency curves. Also, the appearance of multiple orders of diffraction allows more than one incident plane wave to couple to the same Bloch mode. This extra degree of freedom is decisive when trying to optimize the transmission. We illustrate these ideas on a particular rectangular configuration which ensures a strong angular superprism effect, a well collimated transmitted beam, and power transmissions of up to 80%. © *2004 American Institute of Physics.* [DOI: 10.1063/1.1688981]

The superprism effect is the effect whereby the angle of refraction inside a photonic crystal (PC) is seen to vary strongly with the angle of incidence or the frequency of the incident beam. It has been proposed as a means of light deflection for optoelectronic systems such as displays or printers, as well as for communications applications such as multiplexers and dispersion compensators.[1–3]

One problem that continues to frustrate efforts at obtaining practicable geometries and structures for exploiting the superprism effect in photonic crystals has been the tendency of crystals to reflect a large part of the incident light at the interface. In the worst cases the incident light is orthogonal or near orthogonal to the Bloch mode it must couple to, and thus the transmission is negligible. Moreover the superprism effect being a very restrictive problem, one is often forced into a certain configuration by other considerations (band curvature, beam quality, fabrication constraints) and one is left no room for improving the power transmission. Part of the problem is that the geometries that have received the most attention recently have been the highly symmetric square and especially the hexagonal geometry which, because of their high degree of symmetry, allow very little freedom in their design. The hexagonal structure for instance only allows the choice of the hole radius, and the normalized frequency. So we see that the available parameter space is just too small to be able to meet all the requirements, and, at the same time, optimize the transmission. This provides the motivation to study somewhat less symmetrical geometries, such as the rectangular geometry, because they will allow the extra degree(s) of freedom needed to satisfy all the requirements.

In this letter we propose the use of the two-dimensional (2D) rectangular geometry to enhance transmission properties in the angular superprism effect (small change of angle of incidence, large change of transmission angle). It is worth noting that all the ideas presented here are applicable to other geometries and also in the frequency superprism effect (small frequency change, large change of transmission angle).

In this geometry there is one extra parameter, which is absent in the square and hexagonal cases: the aspect ratio of the unit cell (ratio of the longer lattice vector over the shorter). This is already an advantage, because by varying this parameter one can search through a much broader space of possible band structures and iso-frequency curves, allowing more flexibility than in the other, more symmetrical cases. Also, the intrinsic anisotropy of the physical structure results in a more anisotropic band structure, which is favorable for the superprism effect because one is not forced to go into the upper bands to find highly curved iso-frequency curves. But in addition, the horizontal dilation of the lattice introduces multiple orders of diffraction (considering the PC from a one-dimensional grating perspective) which can be used in incidence, meaning that there is more than one option as far as incidence angle, in order to couple to the same bulk Bloch mode. This can be understood by considering the iso-frequency map of the bulk PC.

Figure 1 illustrates the use of the construction line method for a structure with $\omega_n = a/\lambda = 0.203$, $r = 0.25a$ and $x = 2$, in **H** polarization. Here ω_n is the normalized frequency (for a fixed $\lambda = 1.55\mu$), r is the hole radius, a is defined as the length of the shortest lattice vector: $|\mathbf{a}_2| \geqslant |\mathbf{a}_1| = a$, and x is the aspect ratio of the unit cell $|\mathbf{a}_2|/|\mathbf{a}_1|$. The interface, represented by the solid horizontal line of Fig. 1, is therefore the (10) plane. We use a material with an effective index of $n = 3.21$ (following Ref. 4). The dispersion surface used to obtain this diagram was calculated using the MIT-PHOTONIC BANDS package described in Ref. 5.

The construction line method (detailed in Refs. 3 and 6) establishes quantitatively the link between the incident field and the propagative modes in the periodic medium by way of the continuity conditions at the boundary. The large circle represents the modes (plane waves) propagating in the homogeneous medium, and the iso-frequency curves of the crystal the propagative (Bloch) modes in the crystal bulk. The solid horizontal line superimposed on the abscissa is the interface, and the vertical construction lines represent the conservation of the tangential component of the k vector of an incident field at the interface *up to an integer number of lattice vectors*. In this particular case we have three possible

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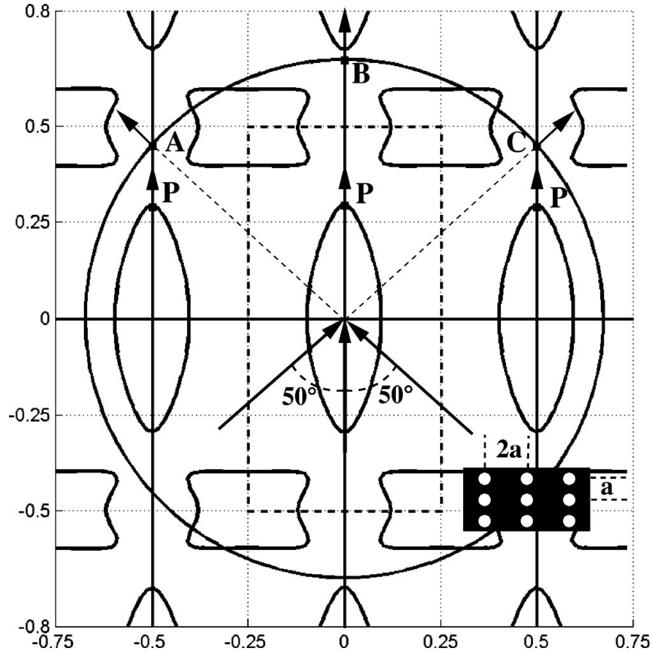

FIG. 1. Iso-frequency curves for the rectangular geometry with $a/\lambda = 0.203$, $r = 0.25a$, $x = 2$, and $n = 3.21$. The solid horizontal line represents the surface of the crystal and the solid vertical lines are the three construction lines, for the same Bloch mode P, giving the three different plane waves A, B, and C. All fields propagate along their respective group velocities (short arrows). The Brillouin zone is indicated by the dashed rectangle.

construction lines, separated by a lattice vector from each other, of course. They relate the same Bloch mode P, to three different plane waves, with wave vectors A, B, and C. The arrows in the figure indicate the directions of propagation of the plane waves and the Bloch mode at P. They follow the group velocities, which are defined as the gradients of the respective dispersion surfaces and are therefore always normal to the corresponding iso-frequency curves.[7]

We see that in order to couple energy into the Bloch mode at P (or into any other mode on the central ellipse shaped curve) we have the choice of three different incident beam directions. We cannot infer anything about the power transmission just by looking at the iso-frequency curve diagram, but we can obtain this information from appropriate finite difference time domain (FDTD) calculations. In Fig. 2 we compare the power transmissions and the angles of transmission for ranges of $\Delta\theta = 3.5°$ around $\theta = 0°$ incidence (the *B* direction) and $\theta = 50.1°$ incidence (the C direction). We see clearly that for the case of the B direction, although the superprism effect is present, the transmission is less than 5%. On the other hand for the C direction the transmission starts at 45% and it reaches 80% for an angle of incidence of 53.5°. The superprism effect is slightly less abrupt, but still very much present: we have a change of 55° in the angle of transmission for a change of 3.5° in the angle of incidence. For clarity we omitted the A direction as its transmission is intermediate (between the B and C directions) and thus less interesting.

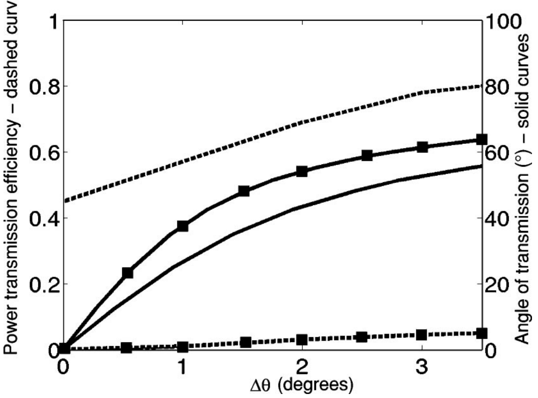

FIG. 2. Transmission efficiency (dashed line) and transmission angle (solid line). Curves with markers are plotted between 0° and 3.5° incidence ($\Delta\theta$ referenced to 0° or the B direction), while those without markers are between 50° and 53.5° ($\Delta\theta$ referenced to 50° or the C direction).

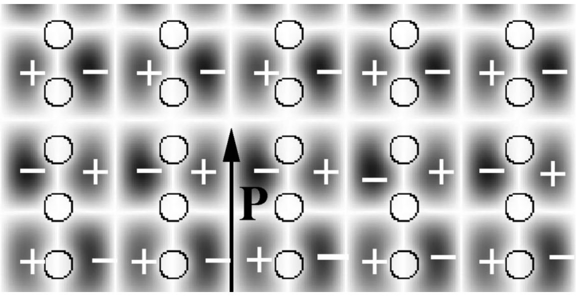

FIG. 3. Spatial **H** field profile for the Bloch mode corresponding to point P in Fig. 1. We see clearly that a plane wave incident vertically would be normal to this mode, which would not be the case for more oblique angles of incidence. Arrow indicates the direction of propagation of the mode.

To understand physically the difference in transmission between the different angles of incidence, neither the iso-frequency curve diagram nor FDTD calculations can provide the necessary insight. One needs to look in more detail at the spatial field distribution of the relevant Bloch modes (in our case the mode at P) to gain a clear insight of the coupling phenomena involved. In Fig. 3 we see the magnitude of the **H** field vector, which is always directed parallel to the holes, because we are in **H** polarization. From the picture it is clear why the B beam is completely reflected: it is directed along a nodal plane of the Bloch mode, and is thus orthogonal to it, which is not the case for the other two beams.

From an overlap integral point of view, the better coupling of the oblique wave can be understood in terms of the field overlap between a plane wave and the Bloch mode in the crystal bulk. In normal incidence it is clear that the overlap integral is zero and the fields are orthogonal, whereas for oblique incidence one may imagine the plane wave as ‘‘lined up’’ in a certain sense, with the rows of pluses and minuses in Fig. 3 leading to larger overlap of the fields, and consequently better coupling. Though clearly far from rigorous or accurate, such mental pictures often help with an intuitive understanding of the (complicated) physical processes.

It is important to note that this improvement in transmission is achieved without modifying the interface at all. What allows for the choice of the most efficiently coupled plane wave is the fact that the elongated shape of the Brillouin zone causes the large, construction circle to overlap the width of not one but three Brillouin zones, giving three orders of diffraction. This essentially means that the boundary conditions now allow the Bloch modes in the crystal to couple with not one, but three modes of the continuum of states in the exterior homogeneous medium. In more symmetrical geometries such as the hexagonal, this also happens, but only for higher frequencies, which are often excluded if one is working in the quasi-2D approximation and the light cone is a problem,[8] or perhaps one is avoiding higher frequencies because the higher bands' tendency to overlap can result, via the construction line method, in poor quality or divergent beams.

In conclusion, we have shown that strongly anisotropic geometries like the rectangular structure provide considerable flexibility in finding appropriate iso-frequency curve configurations. They introduce multiple orders of diffraction which, because of the highly restrictive nature of the technological problem, can prove invaluable in finding practicable superprism configurations. We have demonstrated on a particular rectangular geometry a strong angular superprism effect and a power transmission of up to 80% by using oblique incidence corresponding to the first order of diffraction.

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